

Math club Problem Set #3 Conics:

1. Find the centre and radius of the circle: $4x^2 + 4y^2 + 12x + 16y + 9 = 0$
2. A chord of a circle is a line segment whose endpoints are on the circle. Find the length of the common chord of the two circles whose equations are $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x + 2 = 0$
3. Find the area enclosed in the graph of $x^2 + y^2 = 16x + 32y$
4. Find "r" if "r" is positive and the line whose equation is $x + y = r$ is tangent to the circle whose equation is $x^2 + y^2 = r$ [AHSME]
5. An airline charges \$200 per ticket and sells 40,000 tickets. For every \$10 increase they sell 1000 fewer tickets. How much should they charge to maximize their revenue?
6. Find the smallest possible value of the length of a diagonal of a rectangle with perimeter 36. Prove that your answer is the shortest possible diagonal. Do not just provide an answer:

7. The circumcircle of a triangle is the circle that passes through all three vertices of the triangle. Find an equation whose graph is the circumcircle of a triangle with vertices $(-2,5)$, $(-4,-3)$, and $(0,-3)$
8. Points "A" and "B" are on the parabola $y = 4x^2 + 7x - 1$, and the origin is the midpoint of \overline{AB} . What is the length of \overline{AB} ?
9. Let $f(x) = x^2 + 6x + 1$, and let "R" denote the set of points (x,y) in the coordinate plane such that $f(x) + f(y) \leq 0$ and $f(x) - f(y) \leq 0$. Find the area of "R" [AMC]
10. Find the largest value of "x" for which $x^2 + y^2 = x + y$ has a solution, given that "x" and "y" are real. [ARML]
11. "P" is a fixed point on the diameter \overline{AB} of a circle. Prove that for any chord \overline{CD} of the circle that is parallel to \overline{AB} , we have $PC^2 + PD^2 = PA^2 + PB^2$

12. Square ABCD has sides of length 4, and "M" is the midpoint of \overline{CD} . A circle with radius "2" and centre "M" intersects a circle with radius 4 and center "A" at points "P" and "D". What is the distance from "P" to \overline{AD} AD?
13. Scaling a closed figure in some direction by a factor "k" produces a figure with area "k" times the area of the original figure. Use this fact to explain why the area of an ellipse with major axis of length 2a and minor axis of length 2b is $\pi \times a \times b$
14. An equilateral triangle is inscribed in the ellipse whose equation is $x^2 + 4y^2 = 4$. One vertex of the triangle is (0,1), and one altitude is contained in the y-axis. Find the length of each side of the triangle [AIME]
15. Given that $x^2 + y^2 = 14x + 6y + 6$, what is the largest possible value that $3x + 4y$ can have? [AHSME]

16. Let $f(x) = (x+3)^2 + \frac{9}{4}$ for $x \geq 3$. Find the shortest possible distance between a point on the graph of “f” and a point on the graph of f^{-1} [ARML]

17. Determine the unique pair of real numbers (x,y) that satisfy the equation:

$$(4x^2 + 6x + 4)(4y^2 - 12y + 25) = 28$$

18. The graph of $2x^2 + xy + 3y^2 - 11x - 20y + 40 = 0$ is an ellipse in the first quadrant of the xy -plane. Let “a” and “b” be the maximum and minimum values of $\frac{y}{x}$ over all points (x, y) on the ellipse. What is the value of “a+b” [AMC]